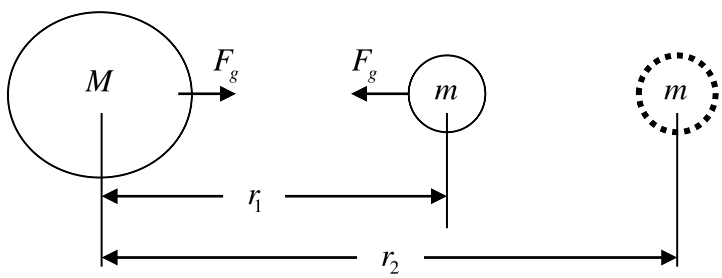
Gravitational Potential Energy

In the previous unit, we developed the equation $E_g = mgh$ in order to calculate the gravitational potential energy of a mass m. This equation, however, is only valid as long as g remains reasonably constant. In other words, this equation is only valid near Earth's surface.

In order to overcome this limitation, we must develop an expression for the gravitational potential energy of a system of any two masses separated by a finite distance.

In the diagram below, the two masses, M and m, are moved from a separation of r_1 to a separation of r_2, by a force that just overcomes the force of gravity between them at every point along the path. They are at rest at both positions.



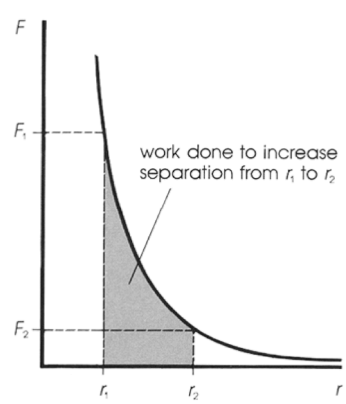
The force of gravitational attraction between the two masses, at any separation distance r, is given by

$$F_g = \frac{GMm}{r^2}$$

To increase the separation of the two masses from r_1 to r_2 requires work to be done to overcome their force of attraction. As a result, the gravitational potential energy of the system increases.

$$W = \Delta E_g$$

The amount of work required to increase the separation of the two masses can be calculated from a graph of Force vs. Separation.



This calculation, which involves calculus, results in the following expression

$$\begin{aligned}\Delta E_g &= E_g' - E_g \\ &= \left(-\frac{GMm}{r'}\right) - \left(-\frac{GMm}{r}\right)\end{aligned}$$

From this expression, we may conclude that

$$\boxed{E_g = -\frac{GMm}{r}}$$

Note:
This equation always produces a negative value. As the separation between the masses increases, E_g increases by becoming less negative. As the separation approaches infinity, E_g approaches zero. In other words, the zero value for gravitational potential energy between two masses occurs when they are infinitely far apart.

Example 1
What is the change in gravitational potential energy of a $60\ kg$ astronaut, lifted from the surface of the Earth into a circular orbit of altitude $400\ km$.

Homework
Gravitational Potential Energy Worksheet

Gravitational Potential Energy Worksheet

1. What is the value of the gravitational potential energy of a 1.00 kg mass on the surface of the earth if the zero of potential energy is taken to be at infinity? ($-6.25 \times 10^{7}\ J$)

2. What is the gravitational potential energy of the moon with respect to the earth if the zero of potential energy is taken to be at infinity? ($-7.7 \times 10^{28}\ J$)

3. What is the change in gravitational potential energy of a 1.00 kg mass that is carried from the surface of the earth to a distance of one earth radius above the surface? ($3.13 \times 10^{7}\ J$)

4. What is the change in gravitational potential energy of a 5.00 kg mass that is carried from the surface of the earth to a distance of 0.25 earth's radius above the surface? ($6.26 \times 10^{7}\ J$)

5. A metal slug is dropped from a height of $0.05r_m$ above the moon's surface. Find the speed with which the slug strikes the moon's surface. ($518\ m/s$)

6. With what initial velocity must an object be projected vertically upward from the surface of Earth, in order to rise to a height equal to Earth's radius? ($7.9 \times 10^{3}\ m/s$)

7. Calculate the change in gravitational potential energy for a 1 kg mass lifted 100 km above Earth's surface. What percentage error would have been made by using the equation $E_g = mgh$ and the value of g at Earth's surface? What does this tell you about the need for the more exact treatment in most normal Earth-bound problems? ($1.0 \times 10^{6}\ J$, 2%)

8. The distance from the sun to Earth varies from $1.47 \times 10^{11}\ m$, at perihelion (closest approach), to $1.52 \times 10^{11}\ m$ at aphelion (farthest distance away).
   a. What is the maximum change in Earth's gravitational potential energy during one orbit of the sun? ($1.8 \times 10^{32}\ J$)
   b. At what point in its orbit is Earth moving the fastest, and what is its maximum change in kinetic energy? (perihelion, $1.8 \times 10^{32}\ J$)